

Fig. 1: Frontal image on Shroud of Turin; note sharpness of image.

### THE VERTICAL ALIGNMENT OF THE FRONTAL IMAGE

# IF THE SHROUD IS DRAPED OVER A RECLINING BODY IN THE SUPINE POSITION, FRONTAL IMAGE FEATURES, TO FIRST ORDER, ALIGN VERTICALLY OVER THE CORRESPONDING BODY PART

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### Introduction

In a previous paper (Ref. 1), we showed that the frontal image on the Turin Shroud, shown in Figure 1, possessed a noteworthy characteristic. The intensity structure of the frontal image could be interpreted as though the Shroud had, in fact, enveloped a body shape and "received" an image by a mechanism that discolored the cloth as an inverse function of cloth-body distance. These studies indicate that image intensity on the Shroud correlates in a one-to-one fashion with cloth-body distance. Another important characteristic of the Shroud image is its sharpness or high resolution, which approaches that of a focussed photograph. This indicates that the discoloration of a given point on the Shroud must be correlated essentially to some unique point on the body. Otherwise, the image would appear blurred due to the overlap influence of many points. These two characteristics suggest that it may be fruitful to consider the Shroud image as a point-to-point mapping between a body and cloth where intensity decreases with distance.

In mathematical terms, we might express this with a distance vector  $D_{pq}$  from some point p on the body to a point q on the cloth where an image of p is recorded with intensity  $I(D_{pq})$ . In order to calculate the intensity distribution of the Shroud image using the relation  $I(D_{pq})$  it is necessary to first specify the vector  $D_{pq}$  for all body/points p. This involves determining both the length and direction of the vectors  $D_{pq}$  which, for the Shroud, is a difficult problem owing to the complexities of the image.

However, the problem can be simplified considerably if the length and direction aspects of  $D_{pq}$  are analyzed separately. Thus, in our previous paper (Ref. 1; pg. 2248), we made what appeared to be a reasonable assumption, based on our cloth drape simulations, that image features aligned everywhere vertically over the corresponding body part. Whether or not this relationship was strictly correct, we assumed it was correct enough to then examine image structure with respect to the length of  $D_{pq}$  only. This study demonstrated a significant degree of correlation for points p associated

with a body shape and q with a cloth surface naturally draping over that body shape.

In this paper, having now established a first order relationship between image intensity and length of  $D_{pq}$ , we return to the directionality problem in light of this finding. Here, we evaluate the directionality properties of  $D_{pq}$ , and thereby test our initial assumptions concerning directionality in Reference 1. If we find agreement, then we will have established a self-consistent, mathematical characterization of the Shroud image. On the other hand, if significant disagreement is found, then we could in principle use the results of this study as new input for directionality and attempt to iterate to a self-consistent characterization of the Shroud image. The goal, of course, is to identify certain symmetries and patterns of the Shroud image that might suggest an image formation mechanism.

Finally, before proceeding, we note that the analyses and conclusions of this paper, because they deal strictly with image structure, are independent of the recent radiocarbon measurement of the Shroud.

#### **Analysis of Image Directionality**

We now consider in detail the problem of determining the directionality of  $D_{pq}$  appropriate for the Shroud image. As in our previous work (Ref. 1), we shall present our discussion from the point of view that the Shroud covered a body. We do not discount the possibility that someone artificially created the image to appear as though the Shroud enveloped a body, but in view of the subtleties of the image discussed both in Reference 1 and in the present paper, we think this hypothesis is incorrect.

Figure 2 shows three paths that information could have taken to reach the cloth. It is conceivable that body surface information might have been mapped along perpendicular paths to the body surface (e), or perpendicular paths to the cloth surface (a), or perhaps along vertical paths between the two surfaces (p). As illustrated in Figure 2, which shows the cross section of a cloth-covered body at the level of the nose and cheeks, each mapping process would obviously produce images of different distortion when the Shroud is laid flat and viewed. Accordingly, it should be possible to deduce from the Shroud image and by reconstructing the implied "burial" configuration of a shrouded body which, if any, of these mappings is appropriate. We have restricted our study to these three mappings because they represent the simplest symmetries associated with the three basic functions of information transfer from a point on the body to a point on the cloth: signal generation — e, channel transfer — p, and signal reception — a. Further, these mappings are consistent with the apparent bilateral symmetry of the Shroud image about the body's longitudinal centerline.



Fig. 2: Paths of possible information transfer from body to cloth at level of nose and cheeks: perpendicular from body surface e, perpendicular to cloth surface a, parallel to axis of symmetry or vertical p.

Let us now form a preliminary evaluation of these three mapping possibilities. Above experimentally determined body and cloth profiles, shown in Figure 3, is a microdensitometer intensity plot of the Shroud image to scale. From this drawing, it is apparent that, of the three mappings discussed above, the vertical direction produces the best correspondence between points on the body and cloth. Projections perpendicular to either the body or cloth surfaces, off the nose in particular, would produce images of significantly greater distortion. Projection perpendicular to the body surface would, in addition, produce a region of "confusion" between the sides of the nose and cheek that is not apparent in the Shroud . image; see Figure 1. From this simple procedure, it would appear that, as assumed in Reference 1, a vertical mapping from the body to cloth would best explain the layout of the image on the Shroud.

However, a more comprehensive and rigorous way to examine the directionality problem would be to start with the given Shroud image and self-consistently compute a threedimensional body surface for each suggested mapping. These surfaces could then be examined for their anatomical correctness. To perform such a computation, it would be necessary to assume both the geometrical configuration of the cloth surface and a reasonable correlation function relating image intensity with distance which must be used for all mappings. If distance is taken to mean the "length" of the mapping vector for each mapping hypothesis, then the problem of computing a body surface for each mapping is mathematically well-defined. \*

Two reasonable criteria for determining a consistent mapping are that the computed body surface (1) be anatomically reasonable and (2) naturally supports a cloth in the geometrical configuration initially assumed for the cloth surface. The latter criterion obviously requires that the assumed mathematical surface used to represent a draping cloth corresponds to the way a cloth drapes over an actual body, a configuration that can be determined a priori by experimental reconstruction over a real body. Figure 4 shows the mathematical reference surface we used in such a calculation to simulate a cloth draping over a human body. Figure 5 shows that a linen cloth, woven in the 3:1 herringbone twill like the Shroud, in general drapes over a body shape like the assumed surface.

We performed the computation described above by first digitizing the Shroud image with a microdensitometer, described in Reference 1, as a sequence of widthwise scans. The measured intensity values were then stored in computer memory along with the corresponding coordinates indicating locations on the Shroud (when laid flat). We then computed a "body" surface by (1) converting image intensity to cloth-body distance for each point on each widthwise intensity scan and (2) projecting that distance from each cloth point in a direction opposite to the mapping vector,  $D_{pa}$ , in the plane of the scan. The computed "body surface" was then constructed by stacking the widthwise "body" profiles in consecutive order such that at natural cloth/body contact points the body profile aligned with the corresponding body part in real life. In our calculation, we accounted for cloth drape effects by relating distances measured along the deformed cloth surface profile to distances measured widthwise along the flat Shroud image, something which was not previously done (Ref. 1; pg. 2248). We then extracted from computer memory the intensity of the image at the equivalent location and converted it to distance, as explained above, to compute the coordinates of the associated body point in space. We constructed the surface to be actual size because we needed to envelop it with our cloth model of the Shroud to test cloth drape characteristics according to the second criterion as discussed above.

The steps of this calculation are shown in Figures 6 through 9.

<sup>\*</sup> An alternative method would be to start with a cloth-covered body and compute an image pattern for each mapping. However, the problem with such a procedure is in deciding what body shape to use. Starting with the Shroud image, computing body shapes for each mapping, and then testing the shapes for anatomical correctness, as we propose, avoids uncertainties in initial conditions for the calculation and allows us to examine the calculated body shapes relative to obtainable physiological data.



Fig. 3: Intensity profile from Shroud image compared in scale to actual body and cloth profiles. Note that the nose region is constrained to a region bounded by a near vertical mapping as opposed to a perpendicular-to-cloth or -body mapping. The two features in the microdensitometer plot on either side of the face correspond to the Shroud hair. These features were not part of the facial profile depicted below and therefore should be ignored. Likewise the apparent bruise on the cheek was not modelled and therefore should also be ignored.

should also be ignored.



Fig. 4: Mathematical reference surface to simulate draping Shroud; compare with Figure 5.



Fig. 5: Linen Shroud model covering a human body shape.



Fig. 6: Shroud image converted to relief in proportion to intensity. Correlation function was selected to make small scale features (i.e., lips, fingers, etc.) anatomically reasonable. The overall appearance of the relief image is somewhat stiff or flat; however, it should be understood that this surface is in actuality a relief map of clothbody distance. As shown in this paper, when this relief is properly plotted from a draping cloth surface, a physiologically reasonable body surface is produced.



Fig. 7: Relief image (left), computed body surfaces assuming vertical mapping (middle) and perpendicular-to-cloth mapping (right).



Fig. 8: Same as Figure 7 except at oblique viewing angle to give sense of 3-D perspective.



Fig. 9: (Top) Linen Shroud model covering vertically mapped statue; compare with Figure 4.



(Bottom) Same Shroud model covering relief statue of Figure 7 left. See Appendix A for discussion.

Figure 6 shows the first step where Shroud image intensity was converted to relief. The transfer function relating intensity to relief was the same as that used in our previous work (Ref. 1) and was chosen to make small scale features (i.e., fingers, lips, nose, etc.) anatomically reasonable using techniques described by German (Ref. 2). Note that waterstains and creasemarks, which have characteristic intensities of their own, are visible in the relief representation. Also, since a fire destroyed the shoulder region of the image, this information does not appear in the relief image.

Appendix A illustrates in detail an actual calculation of body profiles for a scan at the level of the nose. Figures 7 and 8 show calculated body surfaces where computer scans, like that described in Appendix B, are stacked in consecutive order. In particular, Figure 7 shows, along with the Figure 6 relief surface of the Shroud image (left), the computed body surfaces for the vertical mapping where  $D_{pq} = P$  (middle) and perpendicular-to-the-cloth mapping where  $D_{pq} = a$  (right). We did not compute a perpendicular-to-the-body mapping, where  $D_{pq} = e$ , because we expected this to produce a body surface that is generally more distorted than for  $D_{pq} = a$ , because, as illustrated in Figure 2, the body has generally greater widthwise curvatures than the exterior cloth which envelops it. Further, the  $D_{pq} = e$  mapping may be computationally intractable because the body surface must be known a priori in order to calculate the body surface normal. Figure 8 shows the same shapes but from an angle to give a sense of perspective.

The calculated body surfaces of Figures 7 and 8 can be examined for their anatomical consistencies. Per the first criterion discussed above, the statue produced by the vertical mapping, where  $D_{pq} = P$ , appears to be anatomically reasonable, whereas the one for the perpendicular-to-the-cloth mapping, where  $D_{pq} = a$ , seems much too narrow, even emaciated. Figure 9 shows a cloth model of the Shroud draping over the vertically mapped statue. When compared with the cloth surface assumed in our calculation, Figure 4, we see that the vertically mapped statue accommodates a draping cloth in approximately the initially assumed configuration.\*

Thus, based on visual inspection of the computed statues, it would appear as though only a vertical mapping provides a reasonably consistent relationship between a "shrouded" body and

<sup>\*</sup> There is, however, one second order difference that should be noted. As can be seen in Figure 4 and the Appendix B example, the drape of the cloth over the face was assumed to be somewhat less curved than that depicted in Figures 3, 5 and 9 top, which come from actual cloth drape configurations. This was necessary in order for the height structure of the face to be reasonable. The reason for mathematically needing a somewhat more flattened cloth over the face is beyond the scope of this paper, but it could indicate the existence of a chin band or some other factor that was not assumed to produce the draping configurations of Figures 3, 5 and 9 top.

the Shroud image. Such a mapping allows us to start a body surface computation with a cloth surface corresponding to an actual enshrouded body and compute a realistic "body surface" from the given Shroud image. Further, the computed body surface when draped with a cloth model of the Shroud appears to give nearly the same draping geometry as originally assumed. In contrast, an anatomically reasonable body shape does not appear to be derivable by the perpendicular-to-the-cloth mapping, nor by the perpendicular-to-the-body mapping (by inference).

These subjective impressions can be quantitatively substantiated. This is done in Appendix B where it is shown that (1) significant anatomical distortions exist in the Shroud image, (2) these distortions can be reasonably accounted for by an image process that maps body features vertically upward to a cloth draping naturally over a body, and (3) the perpendicular-to-the-cloth mapping between the same body and cloth surfaces would generate image distortions too great to be accounted for by natural anatomical variations. Thus, from this analysis, we conclude that only a vertical (or near vertical) mapping describes the frontal image patterns on the Shroud image in an internally consistent way.

### Sensitivity of Results to Assumptions

Before discussing the significance of this result, we might ask how sensitive our conclusion is to the functional form of the intensity/relief relation that was used. That is, could one "force" the perpendicular-to-the-cloth statue into anatomical consistency by a judicious choice of intensity/distance relation? Certainly, if we could "adjust" the relation to vary in functional form from point-to-point over the body, this might be possible. But that would be very "ad hoc", and we would learn nothing of the possible symmetries of the Shroud image.

The simplest assumption is that a single correlation function of intensity with distance applies globally to the frontal image and this is argued in Reference 1. But as long as we require the correlation of intensity with cloth-body distance to have a single functional form, independent of position over the body surface, then, as the following argument shows, we are restricted to one functional relationship between intensity and cloth-body distance for all mapping possibilities (i.e., p, a, e).

Consider Figure 10 which shows a computer generated relief plot of intensity for the Shroud's facial image. Let us construct an imaginary straight line running longitudinally down this image at its middle (i.e., between the eyes, down over the bridge of the nose, across the center of the lips, and over the center of the chin). As can be seen in Figure 10, the intensity (or, equivalently, relief) varies from a maximum for the entire image (i.e., tip of nose) to nearly that of cloth background (i.e., just below the bottom lip).



Fig. 10: Computer generated relief plot of the intensity values of the Shroud facial image (when the Shroud is laid flat for normal viewing). Regions of high relief (e.g., nose) correspond to intense regions of the image. Note superimposed medial line as referenced in text.

Thus, essentially, the entire range of Shroud image intensities are encountered along this line.

Now consider Figures 4 or 5 showing the cloth in place over a body shape. If we again construct an imaginary medial line longitudinally down the facial region of the cloth, we see that this line corresponds to the apex of the cloth where its local surface geometry is nearly *flat* and *horizontal*, particularly from the nose downward to the chin. This means that along the medial line the vertical mappings, p, and perpendicular-to-the-cloth, a, are indistinguishable from one another. This implies that the same intensity versus cloth-body distance function must be used for both the p and a mappings in order to produce an anatomically reasonable facial structure *along* the midline. But since the range of intensities covering the entire Shroud image are encountered along the midline, it follows that the functional form of the intensity/distance relation is established for *both* mappings along the midline because the mappings there are indistinguishable. Hence, if only one function is to be used for the entire Shroud image, we must use that same function for both mappings p and a because there exist regions where these mappings are functionally identical and in these regions a reasonable anatomical structure must be computed. Accordingly,

we do not have the freedom to vary the correlation function so as to "force" one mapping over the other.

A similar argument applies to the perpendicular-to-the-body mapping, e, because, along the medial line, there are many locations where the body surface geometry is locally flat and horizontal (owing to the bilateral symmetry of the human face). At such locations, the e, p, and a mappings are indistinguishable .from one another. The above arguments apply not only to the face but other areas where the cloth is flat and horizontal (e.g., where the extended medial line intersects the fingers, etc.).

In summary, it is the assumed invariance in the form of the intensity/cloth-body distance function with respect to position across the body surface, coupled with the necessity to compute anatomically reasonable body structure in regions where the Shroud is both flat and horizontal, that quantitatively restricts the choice of intensity/distance relation to the same function for all mappings. Stated alternatively, we cannot use a different correlation function for each mapping in an attempt to compute an anatomically consistent statue in each case because to do so would deform the small scale structure of the resulting statue in regions where the three mappings are degenerate or indistinguishable from one another.

#### **Summary and Conclusions**

Combining the studies of our previous paper (Ref. 1) with those discussed here leads to a simple and internally consistent algorithm for describing the frontal Shroud image in mathematical terms. This algorithm is also explained in Appendix A by a specific example calculation. Consider the body shape, shown in Figure 7 middle, to be in the supine position and enveloped by a cloth draping naturally around it as modelled in Figure 4. At any point on the front or top of this body surface we may define a vector  $D_{pq}$  in the vertically upwards direction with tail at point p on the body and tip at point q on the Figure 4 cloth surface where the vector intersects it. We then compute an image intensity, I, at point q according to a relation  $I(D_{pq})$ , where  $D_{pq}$  is the length of the vector  $D_{pq}$ . According to the studies reported in Reference 1, this relation decreases monotonically with distance to cloth background over a range of approximately 3.7 cm. The explicit form of  $I(D_{pq})$  depends on the format assumed for expressing intensity values, I (e.g., reflectivity at wavelength X. directly on the Shroud, neutral density of a photograph of the Shroud, etc., see Reference 1 for a complete discussion). The correlation function  $I(D_{pq})$  is valid for all points p on the frontal surface. The image pattern so computed will agree with measured intensity values on the frontal Shroud image. The computed image intensities will display a correlation with distance for the frontal pattern. Further, the frontal image will contain distortions consistent with a vertical mapping and be of high resolution. All these characteristics are found on the Shroud's frontal image.

The ability to express the structure of the Shroud image by an algorithm involving a vertical mapping might provide useful insights for determining the Shroud's image formation mechanism. A critical issue is whether the Shroud covered a body shape when the image was formed, as opposed to the Shroud's being the expert handiwork of an artist. The vertical mapping algorithm shows that the complex intensity distribution of the Shroud image can be calculated by a simple relation,  $I(D_{pq})$ , between two complex surfaces, one of which corresponds to an anatomically reasonable body shape and the other to a physically correct cloth surface draping over that body. To our mind, the elegance of being able to describe the Shroud image in this fashion argues strongly that the image was formed while the Shroud enveloped a body as opposed to its being the result of an eye/brain/hand coordination technique of some artist.

It is beyond the scope of this paper to discuss image formation hypotheses that are consistent with a vertical-only mapping through space. Standard processes such as direct contact, diffusion, and radiation (or hybrid combinations thereof) do not act in this manner. Nor is it reasonable to ascribe such a mapping to a hypothetical artist because of the complexity and subtleties involved. Nevertheless, the vertical mapping characteristic should be considered as a basic, first order symmetry of the Shroud image that needs to be explained by any successful image formation hypothesis.

### APPENDIX A Example Calculation of Mapping Statues

Figure 11 shows an example of an actual computer calculation that generated the vertical and perpendicular-to-the-cloth statues. The top curve is an actual intensity scan widthwise across the Shroud face at the level of the nosetip. This intensity curve comes from trace #46 at the tip of the nose in the relief image of Figure 6. The gain of the scanning microdensitometer was adjusted so that intensity values scaled directly to cloth-body distance (Ref. 2); note intensity scale on the left and cloth-body distance scale on the right.

Immediately below the intensity plot is the cloth profile across the nosetip used in the calculation. This curve is a cross section of the mathematical surface used to simulate cloth drape in Figure 4. In Figure 11, the x-axis of the intensity scan was scaled to agree with the distance scale of the cloth curve. We aligned the coordinate systems for the Shroud intensity scan and cloth curve at points C and E, the contact location of the nosetip in both cases.

To compute a point on the body surface, the following procedure was used. Consider point D on the Shroud image. The intensity of that point is



Fig. 11: Example calculation of Shroud statues.

given by B as measured on the left scale. Since the nose is a natural contact point (see Figure 5) we established the zero cloth-body distance level, relative to the right hand scale, at the intensity value of the nose. Thus, the cloth-body distance corresponding to the intensity value of point D is the distance A-B, since the microdensitometer gain was adjusted to plot intensity in units of actual distance.

Next, we located the point D on the given cloth curve at F. This was accomplished by requiring that the distance along the curved surface, E-F, was the same as along the Shroud, C-D. Finally, we determined the body point for the perpendicular-to-the-cloth at G by projecting a distance A-B in a direction perpendicular to the cloth at F. For the vertical statue, we projected vertically downward from F to H the same distance A-B. This procedure was then repeated by computer for all image points along the given intensity scan and for all widthwise scans, head to toe. The results are the body surfaces shown in Figures 7 and 8, middle and right.

It should be noted that the logic used to create the body surfaces is completely reversible. Suppose we wanted to calculate the intensity distribution that a scanning microdensitometer would measure on the Shroud. If we assumed a vertical mapping process and the previously calculated body and cloth curves, we would begin by vertically imaging an arbitrary body point H at the location F on the cloth curve. From this we could refer point F to the Shroud at point D by requiring the distance C-D along the Shroud to be the same as the distance E-F. The intensity of the image at point D is then given as the intensity at contact (in units of distance) minus the distance H-F. The intensity at contact can be calibrated to a single reading of the microdensitometer at the tip of the nose. A simple gain renormalization can then bring the calculated intensity curve into precise correspondence with whatever arbitrary gain setting the microdensitometer happened to have. This procedure, when repeated for all points on the body surface will then generate quantitatively and precisely the complete intensity structure of the Shroud image. A similar procedure can be used for the perpendicular-to-the-cloth mapping if the corresponding statue contour is used, and, again, the Shroud image would be precisely calculated.

The noteworthy point is, as shown in the text, that *only* the vertical mapping surface falls within acceptable anatomical limits for a human body, whereas the perpendicular-to-the-cloth surface does not. The fact that the Shroud image admits such an elegant and straightforward interpretation by computational means, is strong evidence that the image on the Shroud was not created by the hand of man, but by a process that mapped body points vertically upward into the enveloping Shroud cloth.

Finally, we note that the Shroud image intensities could also be calculated precisely by using the relief surface of Figure 6. However, in the context of Figure 11 this would mean that the "cloth" surface would necessarily have to be completely flat since the relief contour is itself merely the measured readings of the microdensitometer. However, neither the relief surface of Figure 7 nor the flat "cloth" surface, shown in Figure 9 right, corresponds anatomically to what is seemingly depicted on the Shroud image, a cloth-covered human body like that shown in Figure 5. From this standpoint such an interpretation is unsatisfying compared to that provided by the vertically mapping process which is consistent with the

Shroud image depiction. In an a priori sense there was no reason to believe that a *consistent*, *simple*, and *physiologically realistic* interpretation of the Shroud image was possible but, as shown in this paper to certainly first order, one has been found via the vertical mapping computation.

## APPENDIX B Quantitative Analysis of Image Distortions and Mapping Hypotheses

As described in the text, the Shroud frontal image appears *qualitatively* to have been produced by a mechanism that mapped body features vertically onto the Shroud, which had been draped naturally over that body. In this appendix, we describe an experiment that *quantitatively* confirms this conclusion. The experiment consists of several parts. First, as initially described in Reference 3, we show that the Shroud image contains geometric distortions that lie statistically outside the normal variation of human anatomy. Then, we show quantitatively that these distortions can be accounted for by assuming that the Shroud draped over a body and that the image transfer was approximately vertical. Finally, we show, again quantitatively, that a mapping perpendicular to the draping Shroud surface overcompensates the distortions in the Shroud image to a degree that lies statistically outside the normal variation of human anatomy.

Our experiment began by selecting a group of ninety-eight male subjects whose heights were  $5'10'' \pm 2.0$  inches (177.8  $\pm 5.1$  centimeters). The purpose was to define the range of anatomical variation appropriate for the Shroud image. This height range was determined by placing subjects of various heights in a cloth model containing an image of the Shroud and determining for what heights correspondence with the image occurred. The cloth model used in our experiment was made of  $345 -\pm 22$  micron thick linen which well approximated the 325 micron thickness of the Shroud and therefore, presumably, its draping characteristics when new. The cloth model's image was scaled according to measurements made by the author directly on the Shroud in Turin.

After ensuring that each subject was generally arranged in the correct position by aligning apparent points of natural contact, the cloth was removed and a photograph of the contorted person, shown in Figure 12, was taken from directly overhead. A grid system under each subject ensured no noticeable lens distortion. It was also interesting to note the apparent physical differences between each subject and the actual Shroud image. There were apparent differences between hip width, arm length, etc.; all these physical features appear to be abnormal on the Shroud.

All ninety-eight photographs were examined to eliminate any subject who had moved from the indicated position because alignment along the head to foot (height) axis was critical in establishing a reference axis which could be compared to width measurements of interest. After reducing the sample population to fifty photographs that aligned in the vertical, teams measured and recorded the distances between selected body points. These measurements and their results are given in Table 1 and annotated in Figure 13.





Fig. 12: Volunteer subjects used in anatomical definition study. Note slight variations in anatomy. These variations give rise to the statistical variations of measurements shown in Table 3, Column 1.

Fig. 13: Location of measurements cited in Tables 1, 3, and 4.

Table 1Average Measurements and Standard Deviations from the Selected Population (in centimeters) (Reference 2*)						
В	$36.3 \pm 1.6$ Width of hips along horizontal level at "V" intersection of hands.					
С	$36.3\pm 1.5$ Width of hips along horizontal level at bottom hand, tip of pointing					
	(first) finger.					
D	$33.6 \pm 1.8$ Width of hips at Man of Shroud's right forearm intersection with					
	body to left forearm intersection with body.					
G	$22.5 \pm 3.1$ Width of knees, horizontal, exterior boundary to exterior boundary.					
Н	$11.8 \pm 1.9$ Width of knees, horizontal, center to center.					
Ι	$13.9 \pm .78$ Width of face at hair/face intersections along horizontal level					
	through center of eyebrows.					
J	$12.6 \pm 1.4$ Width of face at bone structure at edges of eye sockets.					
Κ	$47.9 \pm 2.1$ Length of Man of Shroud's right forearm elbow to tip of middle					
	(second) finger.					
Μ	$33.9 \pm 1.5$ Inner elbow horizontally to inner elbow.					
R	$22.5 \pm 1.4$ Nipple to nipple distance.					

As a check, we compared a subset of our sample measurements with documented 1967 anthropometric data of a fully grown male adult, 5'10" (177.8 cm). Several of the anthropometric measurements were of the same body parameters as the measurements in our sample population; a comparison is listed in Table 2. These measurements are in good agreement and, therefore, support the accuracy of our measurements.

Table 2   Comparative Measurements (centimeters) (Reference 2)				
	Documented Anthropometric Data	Our Sample		
Hip Breadth Right Forearm to tip of finger Face Breadth	38.0 46.5 14.3	$36.3 \pm 1.6$ $47.9 \pm 2.1$ $14.0 \pm .78$		

Next, we performed the same measurements from the Shroud image to compare with our body measurements of Table 1. The photographs used in this determination were a 1931 photograph taken by Enrie and a 1978 photograph by Miller. A 1978 color photograph by Schwortz was also

<sup>\*</sup> In this study we are concerned primarily with distortions in the widthwise direction since the most significant cloth curvatures occur in that direction. We have therefore included only that subset of data entries in Table 1 from Reference 2 that contain widthwise, discernible components. The labels of each entry correspond to those of Reference 2.

used to help define visually certain features. Since our experiment involved measuring image distortions, we had to ensure that cloth stretching and camera distortions were sufficiently small. We evaluated both potential sources of distortion by projecting Miller's 1978 negative image of the Shroud over Enrie's 1931 positive print. The correspondence between these images was remarkably close; we estimated differences  $\Delta \ell / \ell \leq 0.004$  for a length  $\ell$  on the image. Thus, we concluded that cloth-stretch distortions are probably no greater than this value since these two photographs represent two independent occasions of exposition of the Shroud.

We estimated photographic distortion from Miller's 1978 photograph, realizing that the results also apply to Enrie's 1931 photograph by virtue of the above comparison. This was possible because Miller's photograph was a series of three adjacent photographs and slight discrepancies in overlap could be measured. We estimated for photographic distortions,  $\Delta \ell / \ell \leq 0.007$ . The significance of the two distortions is discussed below.

Measurements of the Shroud image are given in the first column of Table 3. Because of fire damage to the actual image on the Shroud, some measurements had to be estimated. For example, the elbow region of the Man of the Shroud's lower forearm has been burned away, thereby hindering accurate measurement of the forearm length. However, we assumed that the elbow was at the burn mark intersection although it could have been further into the burn region, but not shorter. This assumption was also consistent with where the "blood" flow departs from the forearm, and probably indicates where the elbow should be located (see Lavoie's discussion concerning this flow in Reference 4). In this way we were generally conservative in our measurements, tending to underestimate some distances. In addition, the nipple locations, not visible on the Shroud, were nevertheless estimated because the pectoral structure in the Shroud image appears intact from the fire.\*

On the Shroud image, unlike the photographs of the volunteer subjects as in Figure 12, anatomical boundaries are somewhat imprecise owing to intensity fall-off. Accordingly, the Shroud measurements are reported with estimated statistical uncertainties resulting from independent measurements by several persons using 1931 and 1978 photographs with varying contrast qualities. In Column 2 of Table 3 we give the difference between the mean values of the Shroud and subject ensemble (from Table 1), normalized to the mean of the subject ensemble.

<sup>\*</sup> The apparent reason why the nipples are not visible in the Shroud is because intensity correlates only with cloth-body distance, independent of generating surface (i.e., skin, hair, etc.). The skin in the nipple region apparently does not have enough relief structure to generate discernible contrast to be seen. In passing, this argument also calls into question the alleged inscriptions in the "coin" regions. If the nipples fail to have sufficient relief to be visible, how could coin inscriptions nevertheless generate sufficient contrast? In fact, the patterns attributed to coin inscriptions are of high contrast, and if associated with the image, should correspond to over-exaggerated relief structures. In the opinion of the author, there likely are coins over the eyes that can be seen by relief imaging, but the alleged inscriptions are misinterpreted noise patterns that exist over the general image.

Table 3							
Shroud Measurements (in centimeters)							
	1	2	3	4			
	$X_s + \sigma_s$	$\frac{X_{s}\pm -X_{b}}{X_{b}}$	$\frac{(\sigma_{s}^{2}+\sigma_{b}^{2})^{1/2}}{X_{b}}$	COLUMN 2 COLUMN 3			
В	$39.98 \pm 0.65$	0.10	$\pm 0.05$	+ 2.0			
С	$38.00 \pm 1.30$	0.05	$\pm 0.05$	+1.0			
D	$42.63\pm3.41$	0.27	$\pm 0.11$	+ 2.5			
G	$24.85 \pm 1.09$	0.10	$\pm 0.15$	+0.7			
Н	$13.15\pm0.92$	0.11	$\pm 0.18$	+0.6			
Ι	$14.62\pm0.29$	0.05	$\pm 0.06$	+0.8			
J	$14.67\pm0.93$	0.16	±0.13	+1.2			
Κ	$54.00\pm0.61$	0.13	$\pm 0.05$	+2.6			
Μ	$39.46 \pm 1.47$	0.16	$\pm 0.06$	+ 2.7			
R	$22.55\pm0.83$	-0.00	$\pm 0.07$	0.0			
Xs	- mean of Shroud measurements						
$\underline{\sigma}_{\underline{s}}$	- standard deviation of Shroud measurements						
X <sub>b</sub>	- mean of body measurements (see Table 1)						
$\underline{\sigma}_{b}$	- standard deviation of body measurements (see Table 1)						

If we had been able to use actual anatomical measurements from the assumed body form that generated the Shroud image and if the Shroud image boundaries were precise, then any deviations from zero in Column 2 would indicate image distortion of the given Shroud image (as normally viewed, i.e., when stretched flat) relative to the generating body shape. However, since we had to represent the "Man of the Shroud" by a statistical ensemble and the Shroud image boundaries are somewhat ambiguous, we can only assert probable distortion when the (normalized) difference of the mean values of Column 2 exceed the (normalized) statistical uncertainties of the combined subject ensemble and Shroud measurements. If this situation cannot be demonstrated, then image distortions might still be present relative to whatever body generated the image, but undetectable owing to fundamental uncertainties associated with our ignorance of appropriate body anatomy and image definition. Column 3 gives the R.M.S. sum of the standard deviations, again normalized to the mean of the subject ensemble, with which the Column 2 measurements can be compared for statistical significance. In the discussion below, we consider probable distortions to be those where the absolute value of the normalized deviation of the Shroud image from the body ensemble average exceeds the normalized total statistical uncertainty by a factor of about one.

In Column 4 of Table 3 we give the ratio of Column 2 to the absolute value of Column 3, or the number of anatomical standard deviations plus measurement errors that the Shroud image deviates from the anatomical mean. We first note that all the values are positive and many exceed one standard deviation and some more than two. The average number of standard deviations is  $1.41 \pm 0.96$ . This indicates that the Shroud image contains statistically significant widthwise distortions from natural anatomical variation if the Shroud image is viewed while the cloth is laid flat.

We also note that the values in Column 2 of Table 3 are generally much greater than the maximum  $\Delta \ell / \ell = 0.007$  values associated with cloth stretching and photographic aberration; hence these effects cannot account for the image distortions.

In addition to the measurements of Table 3, it should be noted that the Shroud image exhibits a lengthwise distortion of the frontal image compared with the dorsal. The flat frontal image measures head to foot (defined by the blood mark at the foot) at about 6'4" (193.0 cm) while the flat dorsal image head to top of heel at about 5'10" (177.8 cm). This difference likely is the result of distortions induced in the Shroud image by cloth drape over a body.

Having demonstrated that certain aspects of the Shroud image are deformed from normal physical anatomy, we then performed the same measurements on the vertical and perpendicular-to-the-cloth statues, the construction of which is described in the text and in Appendix A. The purpose was to determine if either of these statues, which represent different mappings for a cloth-covered body, are consistent with normal physical anatomy. These results are given in Table 4. The data entries were calculated in the following manner: First, we identified the defining points of the measurements as illustrated in Figure 13, on the relief statue shown in Figure 7 left. Then, by comparing the original scans from which this statue was made with the actual computer generated scans for the vertical and perpendicularto-the-cloth statues (Figure 7, middle and right), we measured the widthwise separations between features. It was noted that measurements for the relief statue of Figure 7 left were systematically less than those made from Shroud photographs (Table 3, Column 1) by  $4.8 \pm$ 4.7%. This variation is not surprising because, as can be deduced from Table 3, Column 1, the relative uncertainty in the Shroud measurements from the photographs was  $4.1 \pm 2.3\%$ . In addition, there appeared to be a general scaling reduction of several percent between the relief statue and the actual Shroud image. Accordingly, in order to properly evaluate the computer calculations it was necessary to normalize the measurements of relief statue of Figure 7 to the mean of the photographic measurements (since these were made from a more fundamental data set). Since the vertical and perpendicular-to-the-cloth statues were derived from the same scan data as the relief statue, they too were normalized by the same normalization factor. The results are shown in Table 4 in terms of the number of anatomical standard deviations plus measurement uncertainties. For convenience in comparison, the similar Column 4 entries of Table 3 are repeated in Column 1 of Table 4. Columns 2 and 3 of Table 4 contain respectively the corresponding values for the vertical and perpendicular-to-the-cloth statues.

From these data we see that the average number of deviations for the vertical statue is -  $0.36 \pm 1.16$  and for the perpendicular-to-the-cloth  $-1.93 \pm 2.11$ . Hence, since only the vertical mapping produces a statue with absolute average distortions less than one standard deviation, we conclude that it alone provides a sensible relationship of image features of a draping cloth over a fully three-dimensional body form. For the perpendicular-to-the-cloth statue, the absolute average deformations are nearly two deviations away from normal anatomy, and thus its associated mapping does not explain the Shroud image in terms of a cloth-covered body.

However, we do note that the variation in the deviations for the vertical statue (1.16) is greater than one, which indicates that possible distortions for certain measurements remain in the vertical statue. Thus, we conclude that some second order effects are present which have not been properly accounted for either by our experimental/computational procedures or which might be intrinsic to the image itself. But on balance, the mean deviations for the vertical mapping of -0.36, being much less than one, singles out this mapping as a satisfactory, first order characterization of the Shroud image.

Table 4Widthwise Distortions for Relief, Vertical and Perpendicular to the Cloth Statues Relative to the Anatomical Mean. Distortions Are Expressed in Number of Anatomical Standard Deviations Plus Measurement Uncertainties and Have Been Normalized to the Mean of Shroud Photographic Measurements of Column 1, Table 3.							
Entry	Shroud	Vertical	Perpendicular				
В	+ 2.0	- 2.8	- 6.2				
С	+ 1.0	- 0.8	- 4.0				
D	+ 2.5	+ 1.7	+ 0.3				
G	+ 0.7	- 0.5	- 1.3				
Н	+ 0.6	- 0.3	- 0.4				
Ι	+ 0.8	0.0	- 3.2				
J	+ 1.2	0.8	- 1.0				
K	+ 2.6	- 0.6	- 1.2				
Μ	+ 2.7	- 0.7	-				
R	0.0	- 0.4	- 0.4				
	$1.41\pm0.96$	- 0.36 ±1.16	- 1.93 ±2.11				

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