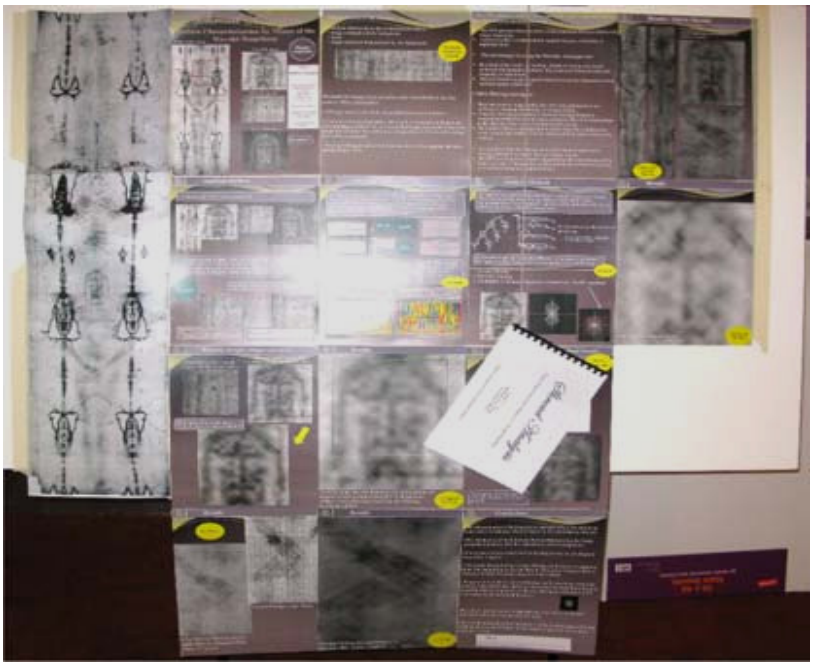
POSTER SESSION: second prize



Shroud Analysis: Texture Characterization by Means of the Wavelet Transform

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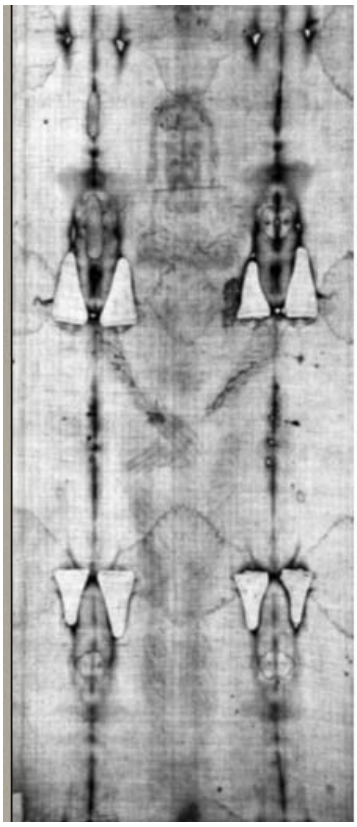


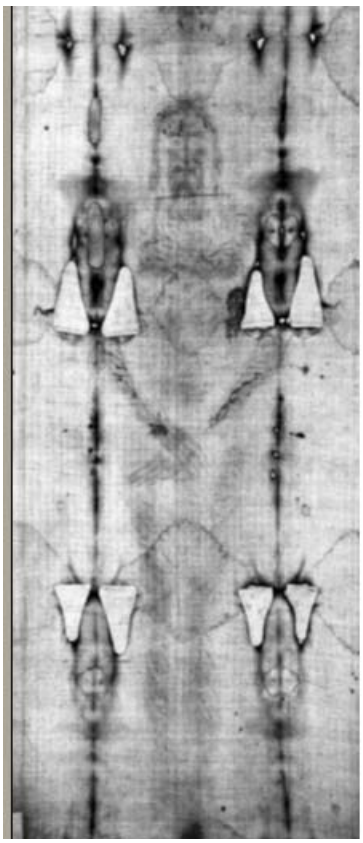
THE SHROUD OF TURIN

Characteristics:

- analysis difficult due to the evident texture effects
- image confused with the background
- burns
- low frequencies of the image impressed

Improvements required for image analysis:

- texture effects elimination
- image denoise
- determination of the frequencies corresponding to defects



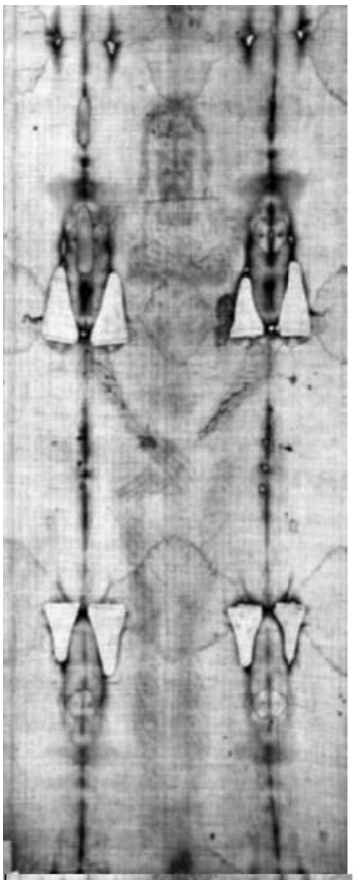
CHOICE of the ANALYSIS METHOD

Possible and most common methods for image analysis, taken into consideration for the texture effect elimination:

- FFT application to determine the periodically repeating defects

- Continuous Wavelet Spectrum of the signal to determine the frequencies corresponding to defects and pyramidal Mallat algorithm to decompose the image and eliminate the coefficients corresponding to defects at the various levels of decomposition

- Filtering techniques such as the Gaussian filter with an appropriate radius and the Median filter



CHOICE of the ANALYSIS METHOD

• the FFT analysis is difficult to use in this case, because:

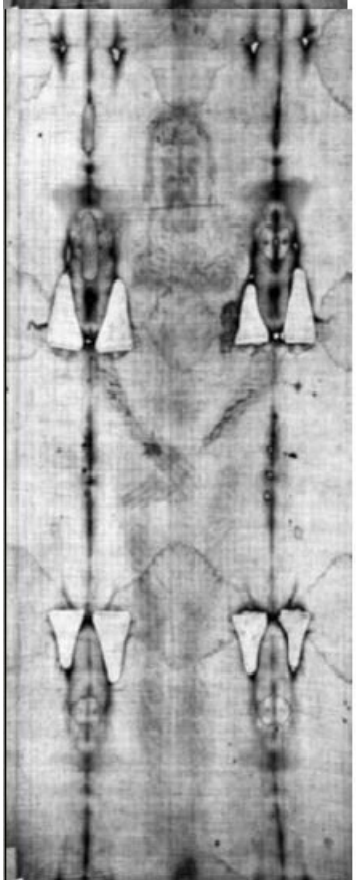
1. The image is characterized by low frequencies, therefore the FFT spectrum does not allow a clear separation between defects and image frequencies

2. Once the analysis window has been chosen, it does not allow a multiresolution analysis because resolution is implicitly fixed

• the advantages in using the Wavelet technique are:

1.By means of the 'scale', or 'scaling', details of various size can be analyzed with adequate resolution. The connection between scale and frequency is maintained, therefore it is always possible to find the position of the image corresponding to a specific range of frequencies.

2.Wavelet Transforms give a good relation between the frequencies found and their spatial collocation, whereas using Fourier Transforms requires a continuous process of choice of the frequencies to be eliminated and control of the results obtained by means of the Inverse Transform.



CHOICE of the ANALYSIS METHOD

• Other filtering techniques:

1.They can be applied after the use of a basic algorithm to eliminate defects if there is the necessity to improve the image quality, but their only application is not sufficient to eliminate the defects (or texture effects).

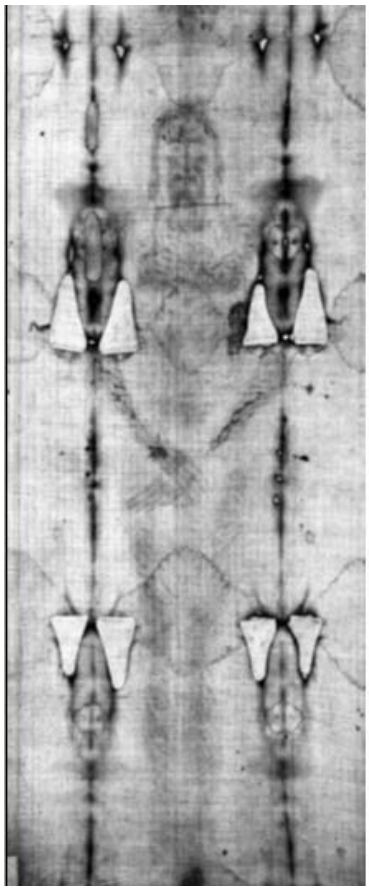
2. Gaussian filtering can be used to eliminate the high frequency components remaining after the denoise process. The main disadvantage is that they can cause a loss of definition of the image already denoised with another process.

3.The median filter is a sliding-window spatial filter, but it replaces the center value in the window with the median of all the pixel values in the window.

• In conclusion, Wavelet filtering seems to be the most appropriate analysis method to be used in the case of Shroud images.

• In particular, the FFT can be applied after having performed the mair. denoise of the image, in order to eliminate the residual defects due to Wavelet filtering.

• The FFT application becomes useful because the wavelet denoise causes a better separation among image frequencies and defects frequencies than for the original image.



ANALYSIS PROPOSED

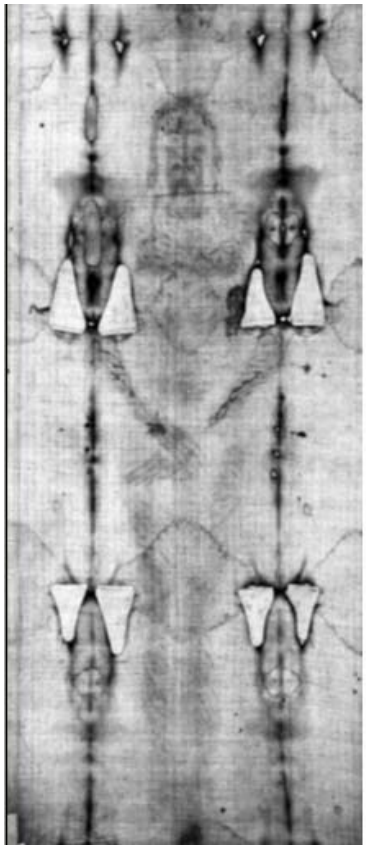
How to get the improvements required for the Analysis (elimination of the texture effect)?

By applying the wavelet filtering technique to some sample images taken from the Shroud.

In fact, after having considered the methods proposed, it is clear that the use of a Continuous Wavelet Spectrum to characterize defects in terms of their frequencies and the application of the Mallat algorithm to denoise the images represent the best process to be used to denoise the images taken into consideration.

This technique allows:

- the elimination of the texture effects
- the determination of frequencies and scales corresponding to defects
- a multiresolution analysis

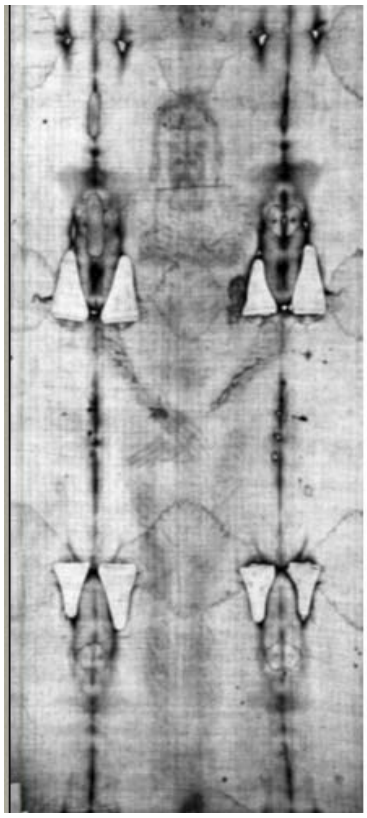


ANALYSIS PROPOSED

The method used can be divided in a series of steps:

1. Determination of the frequencies corresponding to vertical and horizontal defects using continuous wavelet spectrum
2. Application of these frequencies to denoise the Shroud sample images by means of the Mallat pyramidal algorithm
3. Elimination of the residual defects with other filtering techniques, such as:

- the Median filter
- the Gaussian filter
- the elimination of the noisy frequencies directly from the FFT spectrum (its application follows the main denoise procedure based on wavelet)



WAVELET TRANSFORMS

- Wavelet technique maintains the space-frequency relation
- Possibility of analysis with variable resolution
- Good image compression, with high qualities

Continuous wavelet transform:

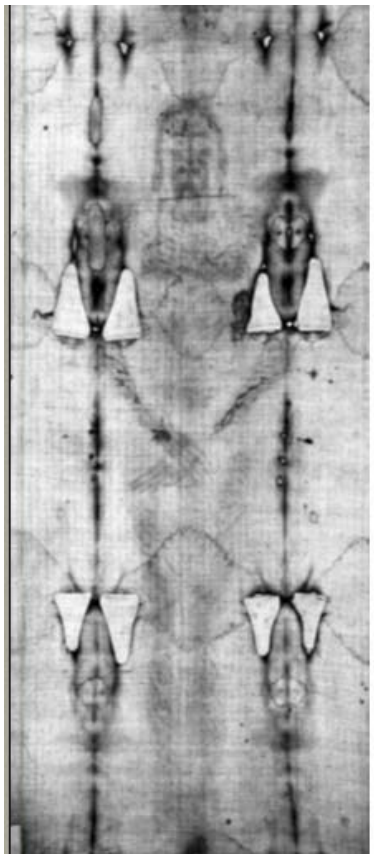
- It is used to determine the frequencies corresponding to defects plotting the Continuous Wavelet Spectrum

Definition: $$Wf(a,\tau) = \int f(t)\psi^{*}_{a,\tau}(x)dx$$

Mother Wavelet: $$\psi_{a,\tau}(x) = \frac{1}{\sqrt{a}}\psi\left(\frac{x-\tau}{a}\right)$$

a = scaling factor
τ = translation factor
$1/\sqrt{a}$ = normalization coefficient

Using different scales means using analysis windows characterized by different dimensions.



WAVELET TRANSFORMS

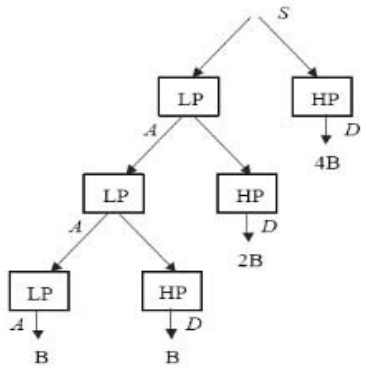
Discrete wavelet transform (DWT):

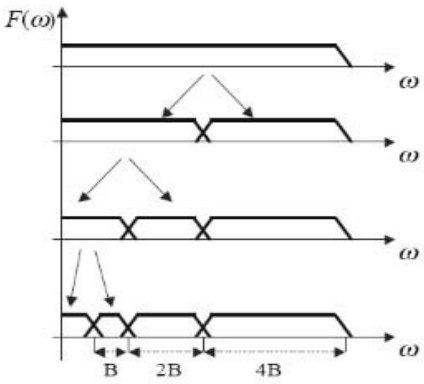
- Used to denoise the images by means of the Mallat pyramidal algorithm

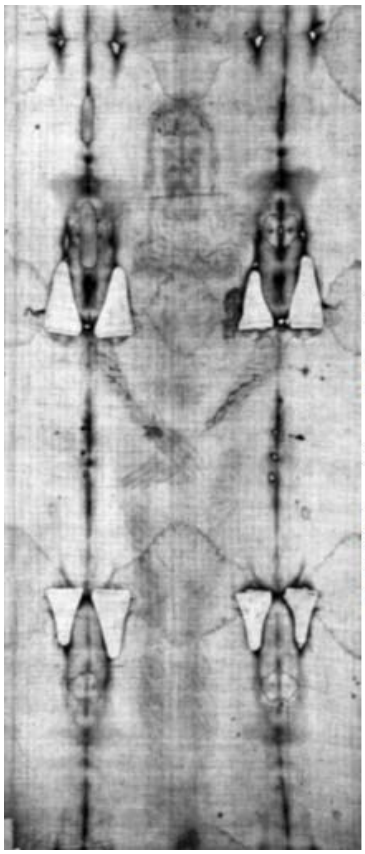
$$\Psi_{j,k}(x) = \frac{1}{\sqrt{a_0^j}}\Psi\left(\frac{x - k\tau_0 a_0^j}{a^j}\right)$$

At every scaling step, the spectrum is divided into:

A: approximation, high pass
D: detail, low pass
S : original signal







MALLAT ALGORITHM

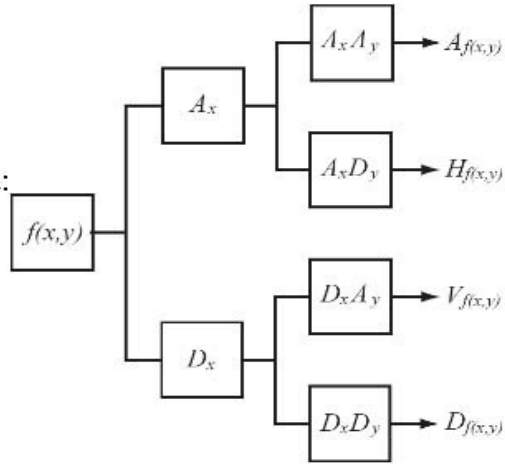
It uses the 2D discrete wavelet transform to decompose the two-dimensional image into various levels of approximation and detail.

Decomposition:

- x direction
- y direction

Four images are generated:

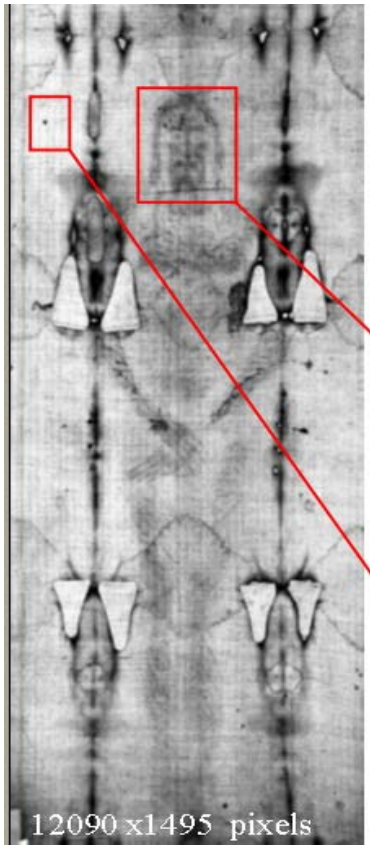
1. Approximation (A)
2. Horizontal detail (H)
3. Vertical detail (V)
4. Diagonal detail (D)



The relation between scale and decomposition level is as follows:

$$a = 2^l$$

a = scaling factor
l = decomposition level



IMAGES ANALYZED

737 x 661 pixels

537 x 591 pixels

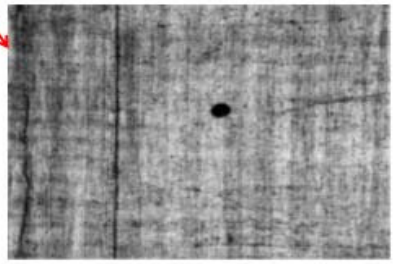
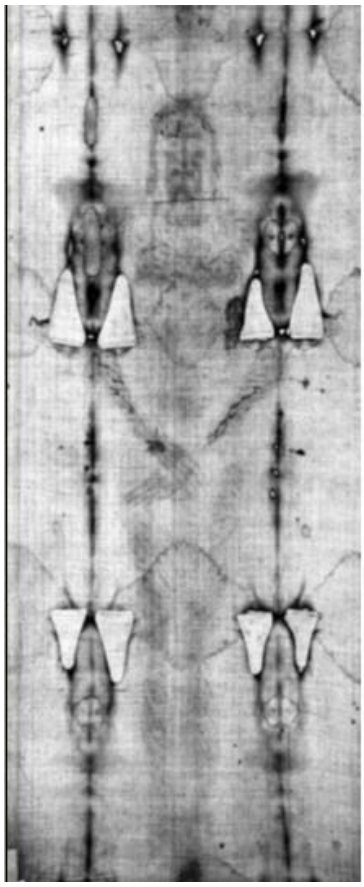


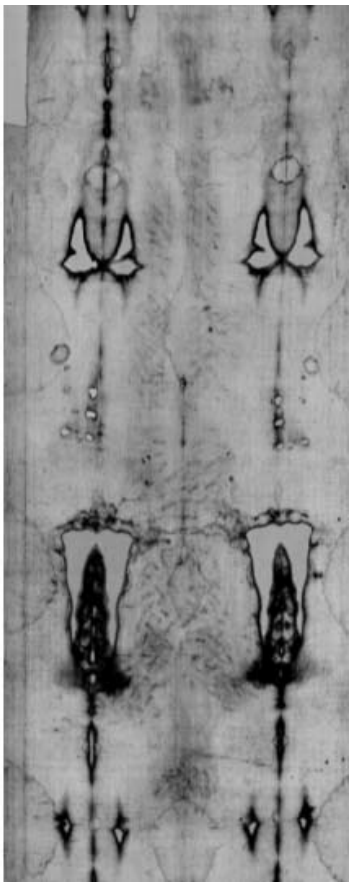
Image of the human face without blood traces

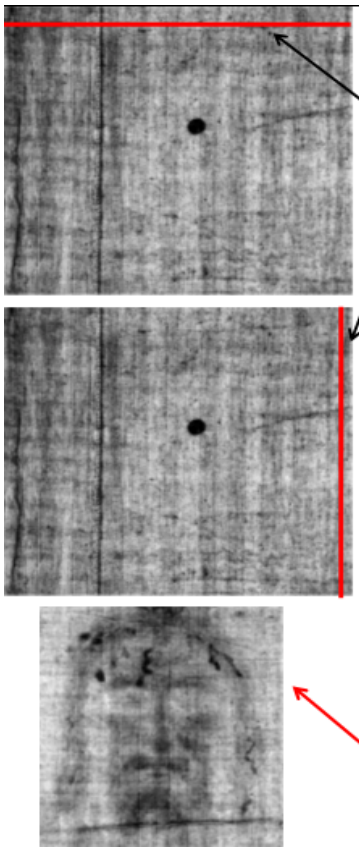
1495 x 1281 pixels



IMAGES ANALYZED

Entire Shroud





WAVELET ANALYSIS OF THE IMAGE OF THE SPOT AND OF THE FACE

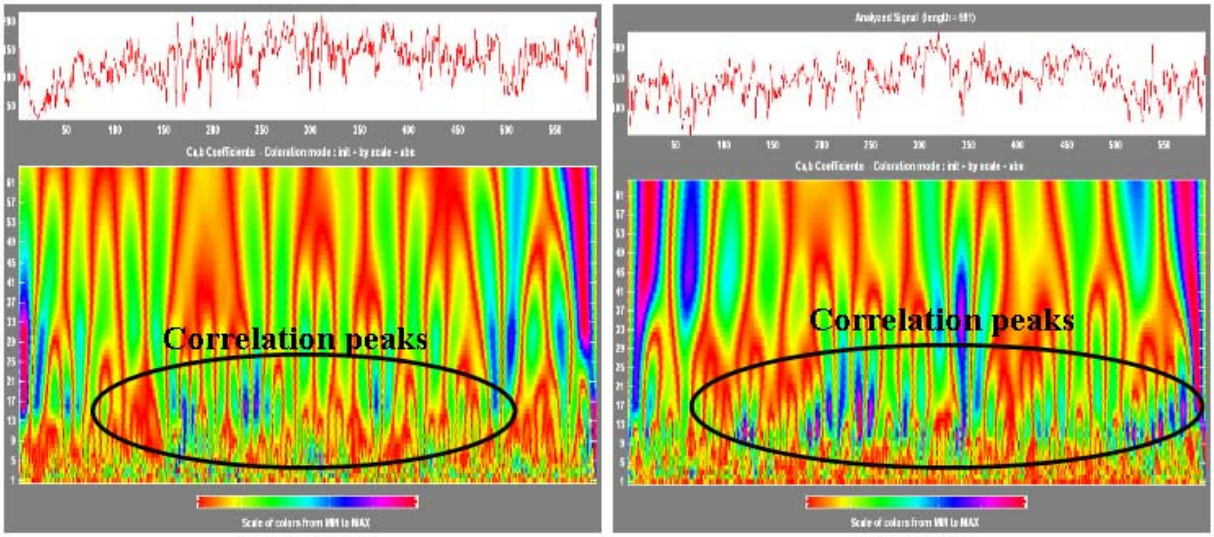
Sampling areas to obtain the gray levels variation

- the one-dimensional signal is sampled in the horizontal and vertical directions
- the sampling areas are appropriately chosen, avoiding areas with burns or other gray levels variations not corresponding to the texture
- it contains information about gray levels variations, due to the texture
- after the sampling, the signal is used to create the continuous wavelet spectrum using ©Matlab Wavemenu
- the spectrum obtained is analyzed to find the correlation peaks, which give information about the frequencies of the defects to be eliminated
- the basic problem is to plot the spectrum using the most suitable wavelet shape
- the frequencies found from these samplings on the left, have been used to denoise both the image of the spot and of the human face, with an acceptable result.

WAVELET ANALYSIS OF THE IMAGE OF THE SPOT AND OF THE FACE

Wavelet Spectrum of the horizontal gray levels variation

Wavelet Spectrum of the vertical gray levels variation



WAVELET ANALYSIS OF THE SPOT AND FACE IMAGES

The frequencies corresponding to defects for the images representing the human face and the spot are as follows:

Frequencies related to Vertical Defects

Wavelet	Scales	Frequencies [1/pixel]
Rbio6.8	5	0.129
Rbio6.8	7	0.092
Rbio6.8	14	0.046
Rbio6.8	25	0.026

Frequencies related to Horizontal Defects

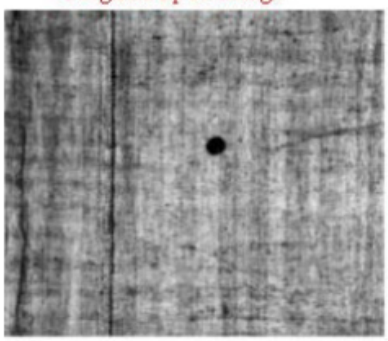
Wavelet	Scales	Frequencies [1/pixel]
Rbio6.8	19	0.028
Rbio6.8	23	0.034
Rbio6.8	9	0.072

These frequencies have been used for the wavelet filtering of two sample images: the spot and the human face with blood spots

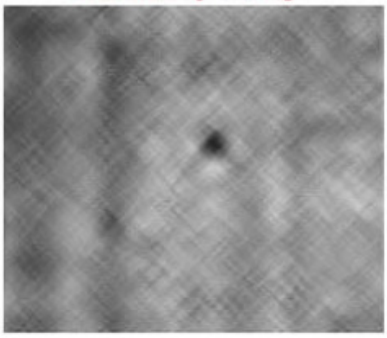
RESULTS

Results after wavelet denoise for the image of the spot

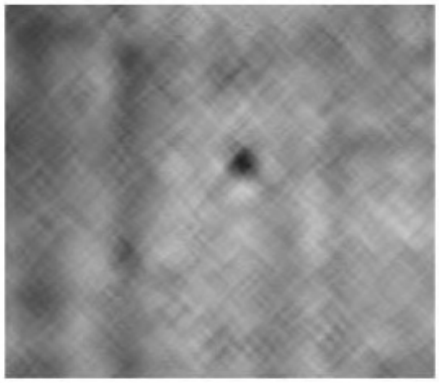
Original spot image



Denoised spot image



Denoised spot image after Gaussian filtering (radius 1.5 pixel)

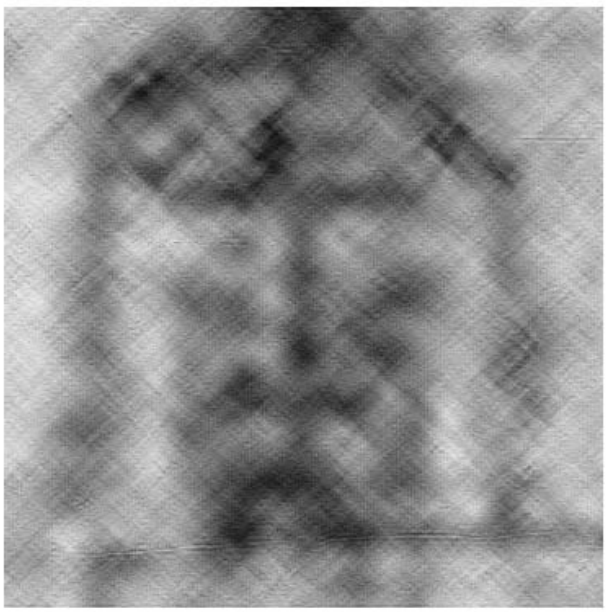


RESULTS

Results after wavelet denoise for the image of the face

Original face image

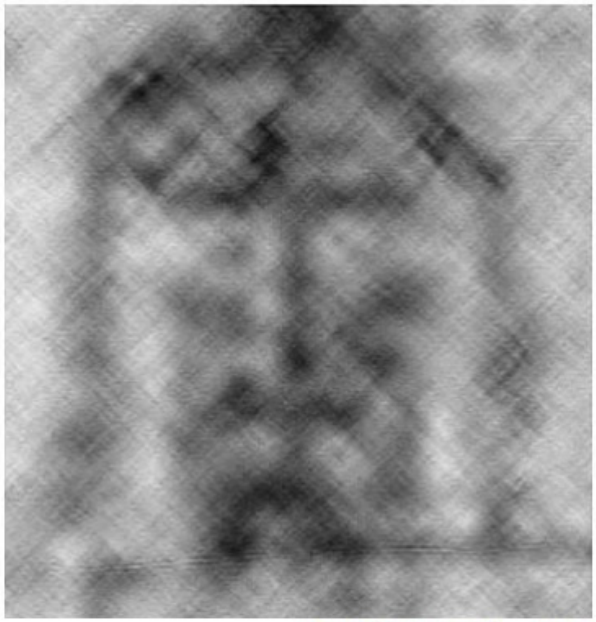
Denoised face image



RESULTS

Results after wavelet denoise and successive filtering for the image of the face

Denoised image of the human face after Median filtering



Denoised image of the human face after Gaussian filtering (radius 3 pixels)

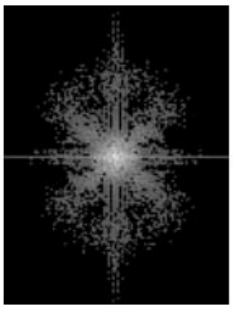
RESULTS ENHANCEMENT

- The results obtained for these two sample images could be enhanced using the FFT spectrum to eliminate the diagonal lines due to the wavelet filtering process

- This is proved by an example applied to the image of the human face

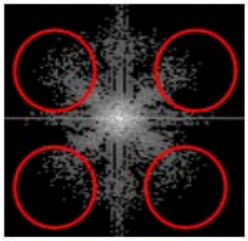
Denoised image after Gaussian filtering (radius 3 pixels)

Its FFT spectrum



RESULTS ENHANCEMENT

Frequencies to be eliminated from the image filtered



The extrema correspond to diagonal lines in the image, the FFT provokes a rotation of 90°

After the elimination of the frequencies highlighted with red circles by means of the FFT →

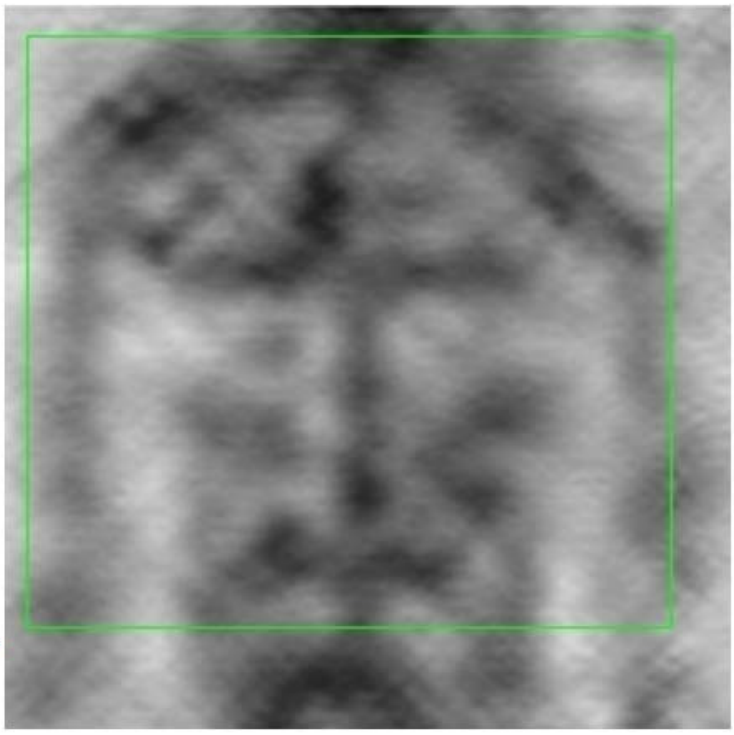
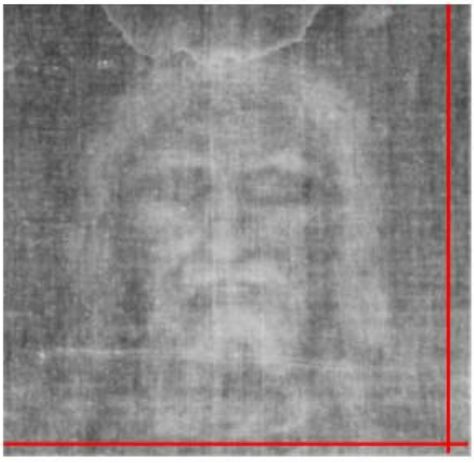


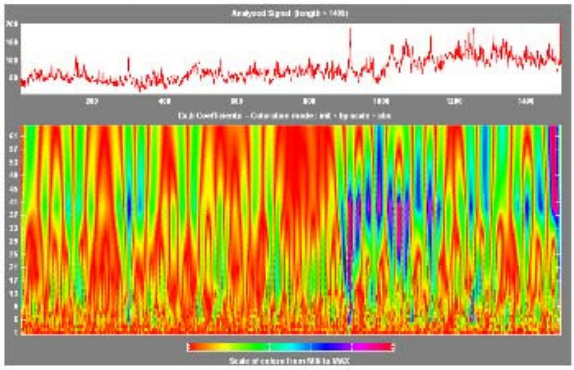
IMAGE WITHOUT BLOOD TRACES

Horizontal Defects →

Vertical Defects →

Sampling areas of this image without blood traces in the horizontal and vertical directions →





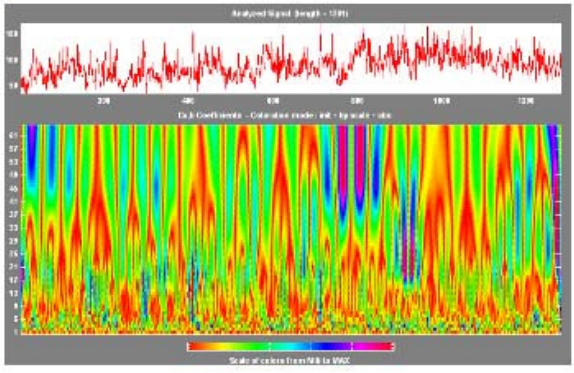


IMAGE WITHOUT BLOOD TRACES

The frequencies corresponding to defects for the image representing the human face without blood traces.

Frequencies related to Vertical Defects

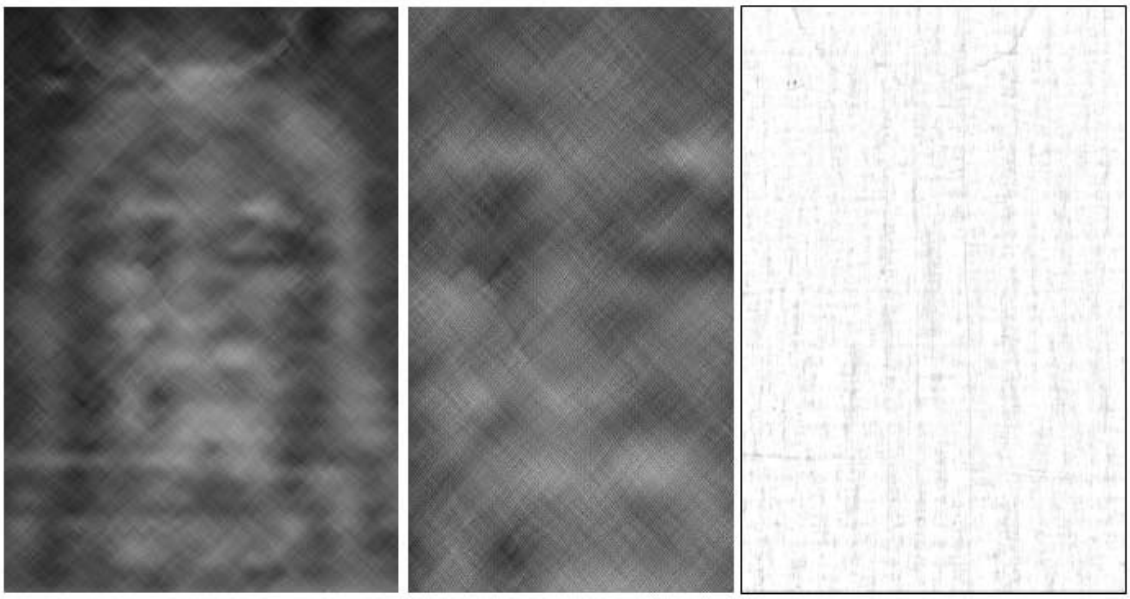
Wavelet	Scales	Frequencies [1/pixel]
Rbio6.8	47	0.014
Rbio6.8	23	0.028
Rbio6.8	18	0.036
Rbio6.8	10	0.065
Rbio6.8	3	0.216

Frequencies related to Horizontal Defects

Wavelet	Scales	Frequencies [1/pixel]
Rbio6.8	41	0.016
Rbio6.8	14	0.046
Rbio6.8	10	0.065
Rbio6.8	5	0.129

These frequencies have been used for the wavelet filtering of the image without blood spots.

IMAGE WITHOUT BLOOD TRACES



After wavelet filtering

Particular showing the diagonal lines and the texture effects left after the main filtering process

Defects eliminated with wavelet filtering

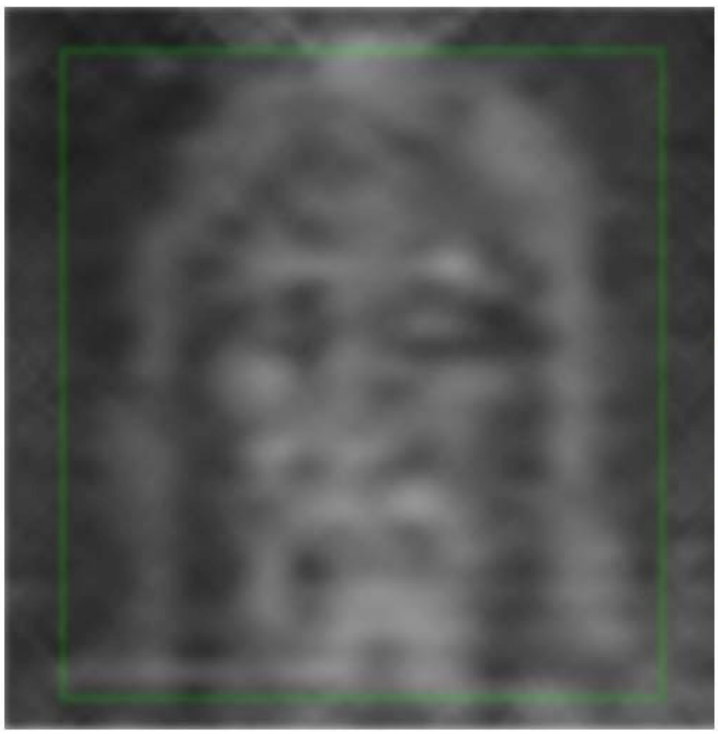
IMAGE WITHOUT BLOOD TRACES

Result after the application of a Gaussian filter (radius 2 pixels)

- The diagonal defects due to the wavelet filtering can be eliminated at least partially by using the FFT, as proved for the other image of the human face containing blood traces

- These effects are only related to the wavelet filtering process, because the other diagonal defects characterized by high frequency and due to the texture have been eliminated with the Gaussian filtering

IMAGE WITHOUT BLOOD TRACES



After the application of a Gaussian filter:

- The FFT spectrum has been used to select the frequencies corresponding to the diagonal lines resulting from the wavelet filtering

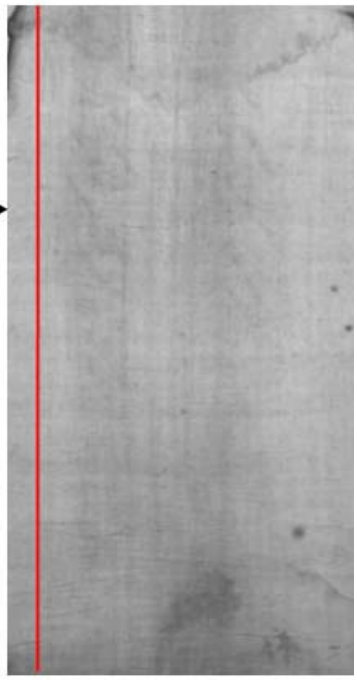
- The diagonal defects caused by the wavelet filtering have disappeared after the final FFT image elaboration

- The green lines correspond to the area elaborated with a specific software for the FFT spectral analysis

ENTIRE SHROUD: SAMPLES

Parts of the Shroud used as samples for:

- vertical defects
- horizontal defects



The choice of these areas for the horizontal and vertical samplings is due to:

- the absence of burns which can represent local effects which are not due to the texture;
- the fact that the horizontal lines are particularly evident in the legs areas whereas the vertical lines result to be visible in the hands area.

ENTIRE SHROUD: FREQUENCIES

The sampling signal seen in the previous page are linked to the frequencies corresponding to vertical and horizontal defects. The hypothesis applied in the study is that the frequencies determined from those samplings are representative of the defects of the entire Shroud. Thus, the same frequencies have been used to denoise every part of the Shroud, even if a particular area was chosen to characterize the defects in terms of their frequencies.

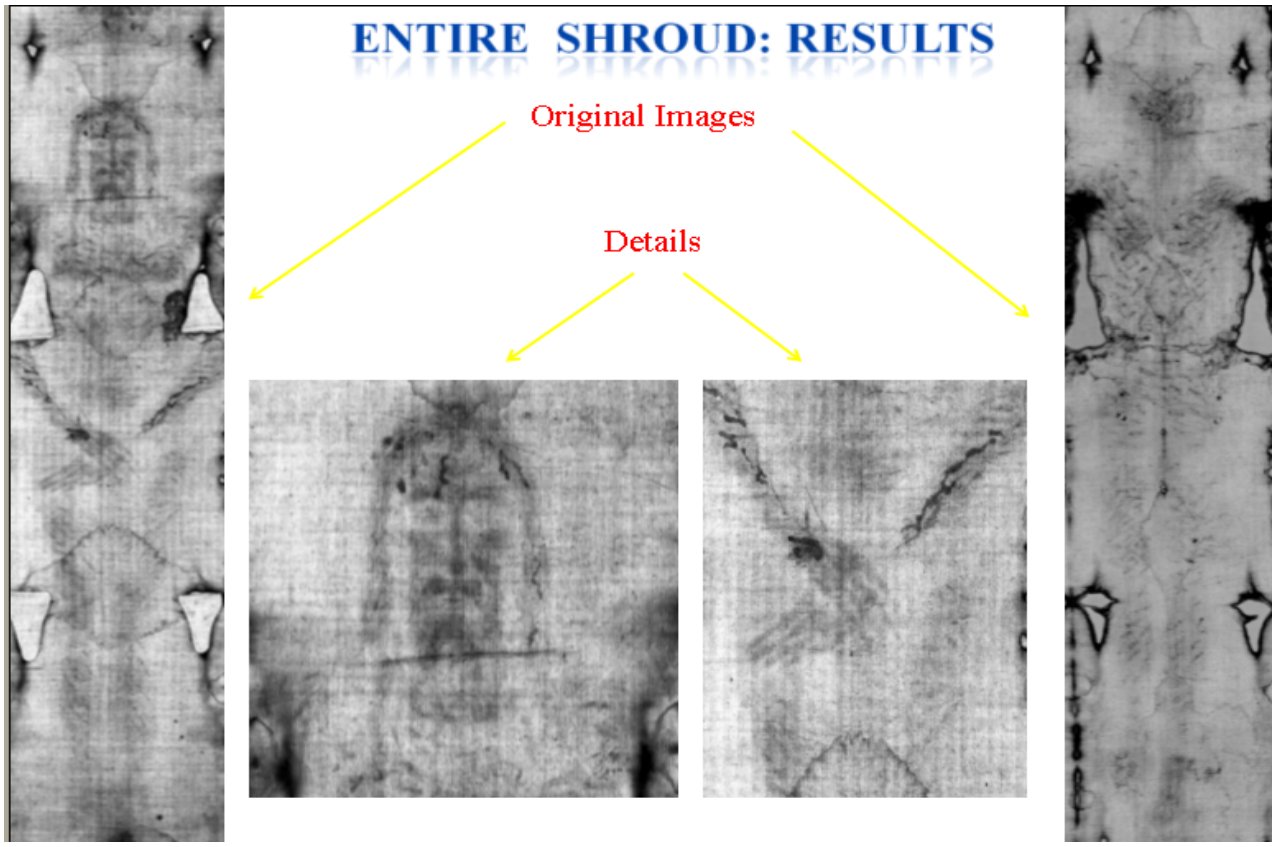
Frequencies related to Vertical Defects

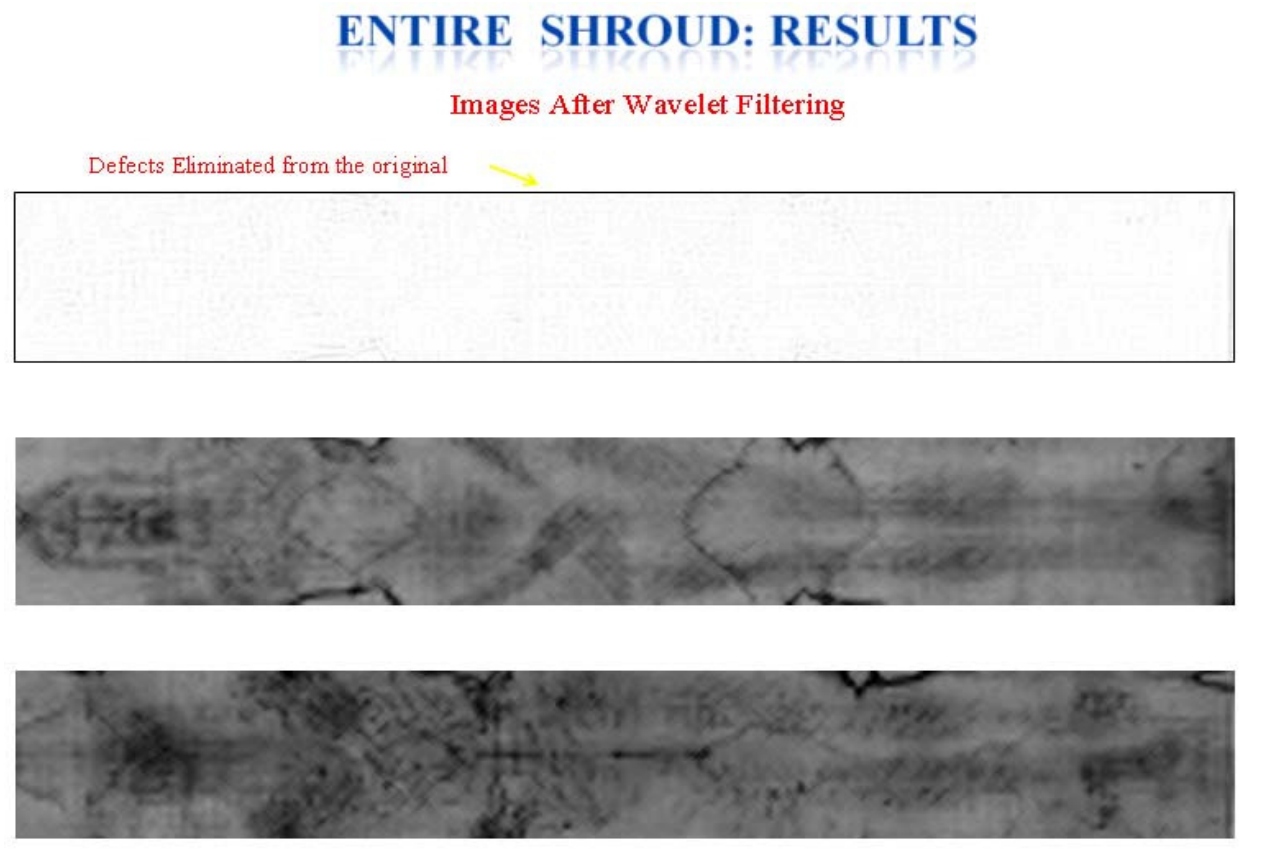
Wavelet	Scales	Frequencies [1/pixel]
Rbio6.8	11	0.059
Rbio6.8	18	0.036
Rbio6.8	28	0.023

Frequencies related to Horizontal Defects

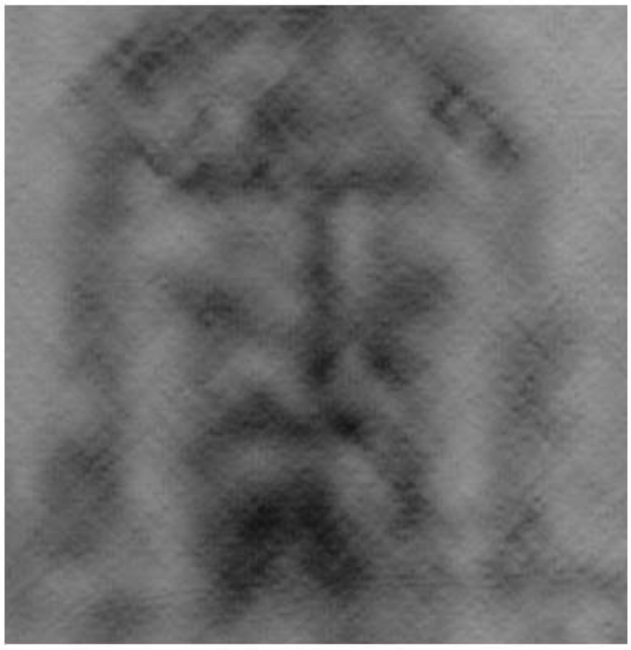
Wavelet	Scales	Frequencies [1/pixel]
Rbio6.8	19	0.034
Rbio6.8	28	0.023

These frequencies have been used for the wavelet filtering of the entire Shroud.

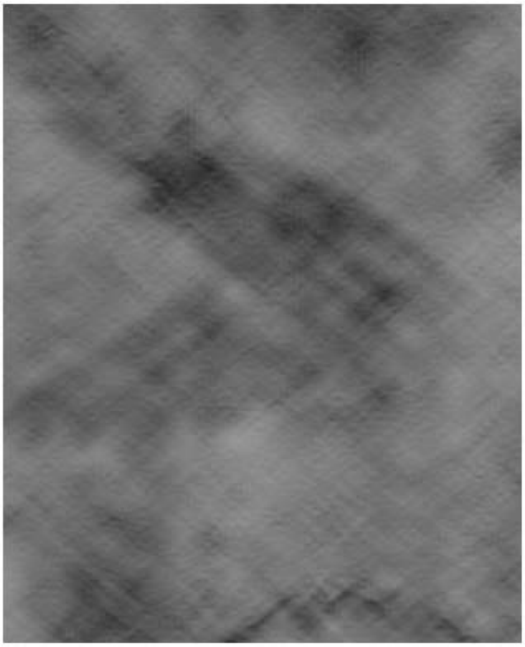

ENTIRE SHROUD: RESULTS
Original Images
Details


ENTIRE SHROUD: RESULTS
Images After Wavelet Filtering
Defects Eliminated from the original

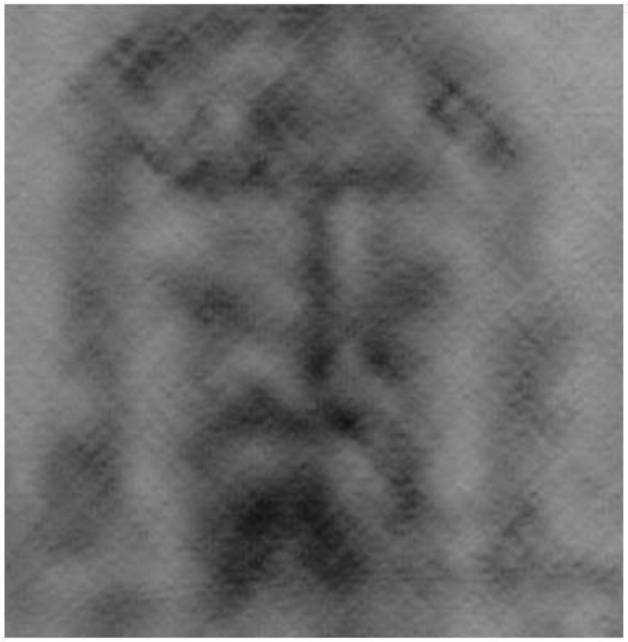
ENTIRE SHROUD: DETAILS



Face and Hands after Wavelet Filtering and Gaussian filtering (radius 1.5 pixel).

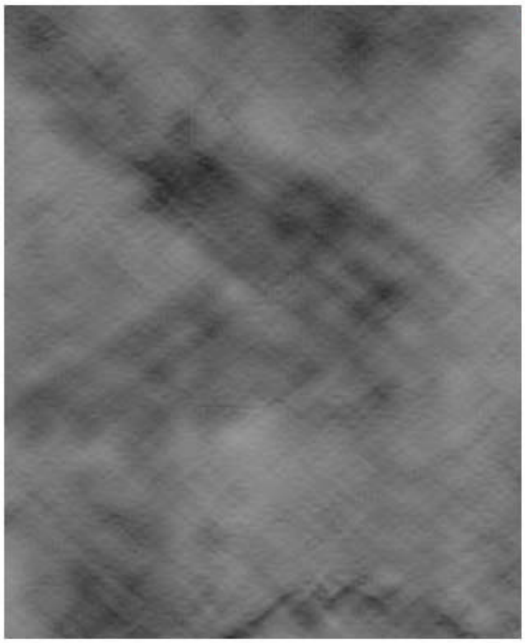


ENTIRE SHROUD: DETAILS



Comparison between the denoised image of the human face and the original image.

ENTIRE SHROUD: DETAILS



Comparison between the denoised image of the hands and the original image

CONCLUSIONS

• The determination of the frequencies corresponding to the defects by means of the Continuous Wavelet Spectrum has proved to be efficient

• The elimination of the frequencies found has been performed using the Mallat pyramidal algorithm

• The elimination of particular combinations of frequencies corresponding to vertical and horizontal defects causes diagonal lines to appear as a noise effect

• The results obtained using wavelet filtering can be improved also applying some other filtering techniques like the Median filter or the Gaussian filter.

• The problem of diagonal defects can be eliminated, or at least reduced, using the Fourier Spectrum of the denoised and filtered image to cancel the highest frequencies of the cross having an inclination of 45° with respect to the horizontal direction.

The yellow lines are meant to highlight the cross visible in the FFT Spectrum of the denoised and filtered image.

• The results obtained with the application of the method proposed in this work have been shown and refer both to the sample images and to the entire Shroud of Turin.

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